

MIT 6.045J - Automata, Computability, Complexity

Lecture 8 - Turing Degrees, Dovetailing, Godel

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In the last lecture we defined Turing degrees, which are basically sets of languages. A Turing degree consists of a set of languages that can all be reduced to another language in the same set using a Turing reduction.

The set of computable languages is called degree 0. The set of languages equivalent to the halting problem is just above degree 0. A natural question to now ask is are there degrees between these two?

It turns out the answer is yes. There are infinitely many Turing degrees between degree 0 and the halting degree. What it means to be “between” these two degrees is that if we have a language L in this intermediate degree, the following relation holds -

$$L \leq_T HALT$$

$$L \not\leq_T \text{degree0}$$

$$HALT \not\leq_T L$$

This means that the language L is reducible to the halting problem. That is, you can build a Turing machine to recognize L given an oracle for the halting problem. However, the language is not reducible to any language in degree 0, and the halting problem is not reducible to language L .